Econ 402
Fall 2007

Problem Set #5
Suggested solutions

5. A destruction of capital.
   (a) The long-run equilibrium is not changed by an alteration of the initial conditions. If the economy started in a steady state, the economy will return to the same steady state. If the economy were initially below the steady state, the approach to the steady state will be delayed by the loss of capital.
   (b) Initially, the growth rate of the capital stock will exceed the growth rate of the labor force. The faster growth rate in capital continues until the steady state is reached.
   (c) The rapid growth rates are consistent with the Solow model's predictions about the likely adjustment to a loss of capital.

8. The golden rule quantity of capital per capita, k^*, is such that $MP_K = zf'(k^*) = n + d$. A decrease in the population growth rate, *n*, requires a decrease in the marginal product of capital. Therefore, the golden rule quantity of capital per capita must increase. The golden rule savings rate may either increase or decrease.

9. (a) First, we need to determine how bN evolves over time:

$$(bN)' = (1+f)(1+n)\, bN$$

Then we just need to redo the analysis of the competitive equilibrium and the steady state as in the book, replacing every N by bN, every $(1+n)$ by $(1+f)(1+n)$, and every n by $f+n$. The new steady-state per efficiency unit capital is then

$$k^{**} = \frac{szf(k^{**})}{(1+f)(1+n)} + \frac{(1-d)k^{**}}{(1+f)(1+n)}$$

All aggregate variables then grow at the rate of $f+n$, while per capita aggregates grow at the
rate f.

(b) An increase in f increases the growth rate of per capita income by the same amount, as f is its growth rate. This happens because the exogenous growth in b raises instant capital and income for everyone without a need to invest in capital.

10. Production linear in capital: $\frac{Y}{N} = z\frac{K}{N} = zf(k) \Rightarrow f(k) = k$

(a) Recall Equation (20) from the text, and replace $f(k)$ with k to obtain:

$$k' = \frac{(sz+(1-d))}{(1+n)}k$$

Also recall that $\frac{Y}{N} = zk \Rightarrow k = \frac{1}{z}\frac{Y}{N}$ and $k' = \frac{1}{z}\frac{Y'}{N'}$. Therefore:

$$\frac{Y'}{N'} = \frac{(sz+(1-d))}{(1+n)}\frac{Y}{N}$$

As long as $\frac{(sz+(1-d))}{(1+n)} > 1$, per capita income grows indefinitely.

(b) The growth rate of income per capita is therefore:

$$g = \frac{\frac{Y'}{N'} - \frac{Y}{N}}{\frac{Y}{N}} = \frac{(sz+(1-d))}{(1+n)} - 1$$

$$= \frac{sz-(n+d)}{(1+n)}$$

Obviously, g is increasing in s.

(c) This model allows for the possibility of an ever-increasing amount of capital per capita. In the Solow model, the fact that the marginal product of capital is declining in capital is the key impediment to continual increases in the amount of capital per capita.